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DERIVATION OF MOS-UNIQUE CONTINUATION RATES.(U)

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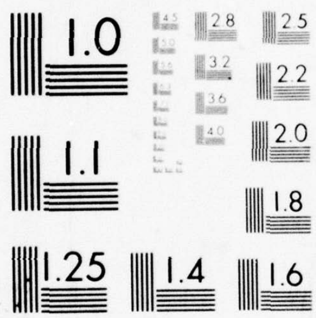

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Derivation of continuation rates at the MOS level of detail is complicated by promotions which involve a change in MOS and by MOS reclassifications within grade. In the methodology developed, the effects of the two factors are accounted for by using a directed graph to represent the total force, where the nodes represent populations for each MOS/Grade pair and the arcs quantitatively represent personnel flows between nodes.

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DERIVATION OF MOS-UNIQUE CONTINUATION RATES

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DERIVATION OF MOS-UNIQUE CONTINUATION RATES

SUMMARY

To manage Army personnel effectively, accurate predictions of the future personnel status of each military occupational specialty (MOS) are essential. The Army has developed several deterministic models capable of projecting personnel strengths by MOS up to 4 years in time. In making projections, the models require continuation rates (probabilities that individuals will remain in service) for each year of service. The continuation rates used by these models have been derived from the total Army population; thus, predictions have been accurate only at the total force level and not at the individual MOS level as required. However, accurate predictions could be made at the MOS level if the continuation rates were computed at the MOS level.

Derivation of continuation rates at the MOS level of detail is complicated by two factors: (1) promotions involving a change in MOS, and (2) MOS reclassifications within grade. The latter is normally used to balance MOS in which assets greatly exceed or fall short of authorizations. In the methodology developed, the effects of the two factors are accounted for by using a directed graph to represent the total force, where the nodes represent populations for each MOS/Grade pair and the arcs quantitatively represent personnel flows between nodes.

To develop pure continuation rates at the MOS level, the Backward Star Traversal Algorithm is used to remove the promotion and reclassification effects. Convex combinations of the pure rates are then computed for each node of the graph according to the composition of the node. The convex combinations are used as estimates of the continuation rates by MOS and grade for each year of service.

CONTENTS

DERIVATION OF MOS-UNIQUE CONTINUATION RATES

CHAPTER 1
INTRODUCTION

1-1. BACKGROUND. Effective personnel management requires the Army to be able to predict accurately personnel shortages by Military Occupational Specialty (MOS). There are over 400 enlisted MOS in the Army and the skills related to each MOS tend to be narrowly defined. The skill definitions in many cases are directly related to specific grades and, as a result, promotions frequently involve changes in MOS. An added complication is introduced by MOS reclassifications within grade which results from attempts to balance chronic surpluses and shortages.

1-2. PROBLEM. The Army maintains data on the historical populations of MOS by both grade and year of service; however, in the past the Army has been unable to derive MOS-unique continuation rates* from these data because of the changes of MOS. The difficulty has been that true losses to an MOS in the Army could not be distinguished from apparent losses caused by changes of MOS. In the absence of MOS-unique continuation rates, average rates based on the total Army population have been applied at the MOS level to project the force over time. This has caused predictions to be accurate only at an aggregated level and not at MOS level as required. The Personnel Inventory Analysis/Year of Service (PIA/YOS) model[1/] is used by the Army to project the force over time. The model requires two key sets of input data: continuation rates by MOS/grade/year of service and "feeder patterns" which specify the direction and relative magnitude of personnel flows. The model first computes updated populations by grade and MOS using the continuation rates and then simulates promotion and reclassification according to the feeder patterns. Since promotion is modeled explicitly, the continuation rates used must reflect only true losses to the Army. The average continuation rates based on the total Army population as previously used do not accurately reflect true losses at the MOS level.

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*Continuation rates are defined as probabilities: the continuation rate for year of service i is the probability that an individual in his i^{th} year of service will remain in service at least one more year.

1-3. PURPOSE. This paper describes a technique by which more precise MOS-unique continuation rates needed by the PIA/YOS model can be derived. The PIA/YOS feeder patterns, which also serve as input to the PIA/YOS model, are first restructured as a directed graph. The graph is then used in conjunction with historical data on MOS populations to derive continuation rates which are unique by MOS, grade and year of service and yet exclude the effects of promotion. These rates are then substituted for the previously defined rates based on the total Army populations and used as input to the PIA/YOS model.

CHAPTER 2
DERIVATION OF MOS-UNIQUE CONTINUATION RATES

2.1 GENERAL. In the directed graph formulation, each combination of an MOS and grade is considered as a node and both promotion and reclassification are represented as arcs. Before generalizing this concept, consider the system shown in Figure 2-1 which has only two MOS, A and B, and two grades, E1 and E2.

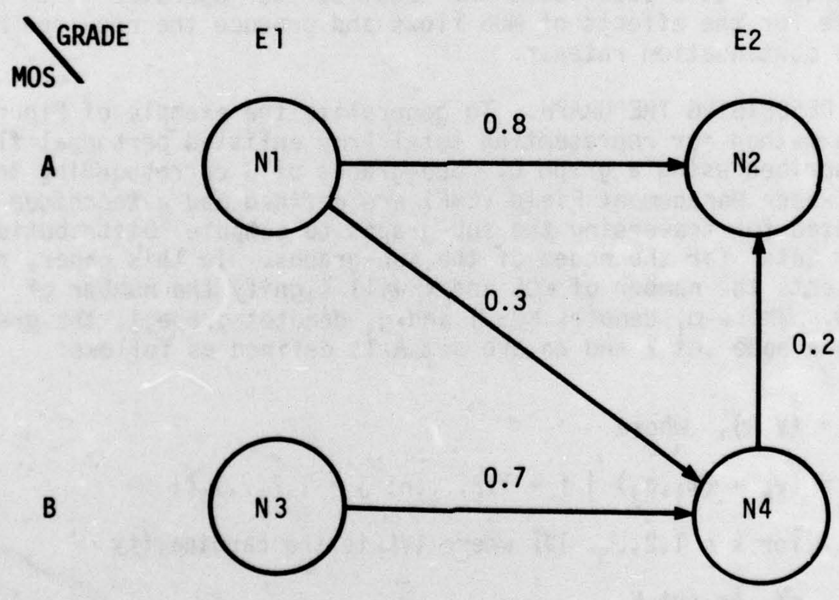

Figure 2-1. A Notional Example Illustrating Node and Arc Information

In the directed graph, the arcs represent potential flows between nodes; however, the amount of the flow is determined by the unfilled requirements at the receiving node. In Figure 2-1, some unfilled requirement may exist at node N2 after attrition. If a requirement does exist, 80% of that requirement will be filled from node N1 and 20% from node N4. Calculation of continuation rates for even the simplified example is not straightforward. The sub-populations at nodes N1 and N3 can be categorized easily since these are entry level grades. Nodes N2 and N4 are not so easily described. Although the sub-population of N2 currently holds MOS A and grade E2, at least part of that sub-population consists of people who originally held MOS B. To further complicate matters, some of the people serving at node N2 who originated in MOS A have also served in MOS B having moved on the arcs from N1 to N4 and then N4 to N2. The "Backward Star Traversal Algorithm" (BSTAR) developed in this paper uses the negative star operator [2] to compensate for the effects of MOS flows and produce the required MOS-unique continuation rates.

2-2. DESCRIBING THE GRAPH. To generalize the example of Figure 2-1, a method for representing total Army enlisted personnel flows is described using a graph G. Sub-graphs of G corresponding to each Career Management Field (CMF) are defined and a technique is presented for traversing the sub-graphs to compute "Distribution Vector Sets" for the nodes of the sub-graphs. In this paper, n represents the number of MOS and ℓ will signify the number of grades. Where m_i denotes MOS i and g_j denotes grade j, the graph G with a node set V and an arc set A is defined as follows:

(1) $G = (V,A)$, where

(2) $V = \{v_k = (m_i,g_j) \mid i = 1,2,\ldots n;\ j = 1,2,\ldots,\ell\}$

for $k = 1,2,\ldots |V|$ where $|V|$ is the cardinality of the set V

(3) $A = \{a_x = (v_s,v_t,p_{st}) \mid s \neq t;\ s = 1,2,\ldots, |V|\ ;\ t = 1,2,\ldots,|V|;$

$0 \leq p_{st} \leq 1\}$ for $x = 1,2,\ldots, |A|$

Because grades E1 through E3 are frequently considered together, the full range of enlisted grades (E1-E9) are modelled with ℓ equal to 7. The number of MOS could vary from a low of 400 to an upper bound of 600 depending on the time frame of the data. Attempting to solve the problem at the total Army level presented two major computer-related problems. First, computational time is directly proportional to the number of MOS and it was estimated that a solution of a complete graph would require from 4 to 6 CPU hours. Secondly, core requirements would exceed the limits of most computing facilities and could require segmentation of the problem. Fortunately, these problems can be avoided because MOS are currently managed and controlled within disjoint Career Management Fields. If λ is the number of CMF's, sub-graphs $\bar{G}_i$ of G can be defined in the following manner:

$$(4)\quad \bar{G}_i = (\bar{V}_i, \bar{A}_i)$$

$$(5)\quad \bigcup_{i=1}^{\lambda} \bar{G}_i = G; \bar{G}_i \cap \bar{G}_j = \phi \ \forall \ i,j \ni i \neq j$$

where Φ is the null set

$$(6)\quad \bigcup_{i=1}^{\lambda} \bar{V}_i = V; \bar{V}_i \cap \bar{V}_j = \Phi \ \forall \ i,j \ni i \neq j$$

$$(7)\quad \bigcup_{i=1}^{\lambda} \bar{A}_i = A; \bar{A}_i \cap \bar{A}_j = \Phi \ \forall \ i,j \ni i \neq j$$

Neither G nor the sub-graphs $\bar{G}_i$ are directed graphs as defined. However, in order to conform to the feeder patterns, G and therefore $\bar{G}_i$ must be considered directed graphs of a special nature. If $v_s = (m_i, g_s)$ and $v_t = (m_j, g_t)$, then the feeder patterns require that:

$$(8)\quad g_t \geq g_s \ \forall \ a = (v_s, v_t, p_{st}) \in A$$

(9) Circuits are not allowed

Constraint (8) requires a left-to-right and up-or-down ordering corresponding to promotion or within grade MOS reclassification and (9) is needed to insure that the algorithm will terminate. Figure 2-2 illustrates the basic arc types allowed by the constraints.

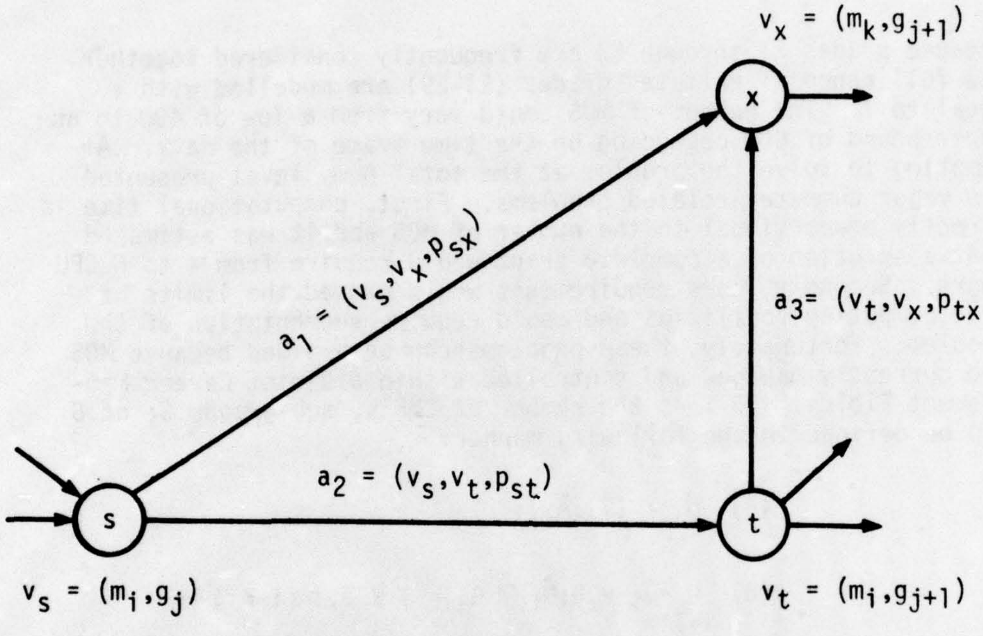

Figure 2-2. Arc Types in the Directed Graph

In Figure 2-2, arc a_2 is a promotion arc since the MOS remains the same, but the grade increases. Arc a_3 is a within grade reclassification arc since the grade remains the same and the MOS changes. Arc a_1 is a promotion with reclassification arc since both the grade and MOS change. To traverse the sub-graphs, the BACKWARD STAR OPERATOR is defined:

(10) $\partial\{v_t\} = \{a \varepsilon \overline{A}_i \mid a = (v_x, v_t, p_{xt}) \ \forall \ v_x \varepsilon \overline{V}_i\}$

Thus $\partial\{v_t\}$ identifies all arcs which terminate at node v_t. In Figure 2-2, $\partial\{v_x\} = \{a_1, a_3\}$ and $\partial\{v_t\} = \{a_2\}$

2-3. DESCRIBING PRIOR MOS FLOWS. The set $D = \{D_1, D_2, \ldots, D_k\}$ where $k = |V_i|$ is defined to be a "Distribution Vector Set," where the elements in D are in one-to-one correspondence with the elements (nodes) in V_i. D is an n-tuple of the form $D_x = (d_1, d_2, \ldots, d_n)$ where n is now the number of MOS in the CMF. The purpose of the distribution vector is to summarize information on prior flows to a given node. The entries in D_x should contain the fraction of node v_x that originated in MOS 1, MOS 2, ..., MOS n. Thus $0 \leq d_k \leq 1$

for $k = 1,2, \ldots, n$ and $\sum_{k=1}^{n} d_k = 1$. For computational ease, an n by ℓ matrix Γ is established and the node set $\overline{V}$ is mapped into Γ such that

$$\Gamma_{i,j} = \begin{cases} 1, \text{ if } v = (m_i, g_j) \varepsilon \overline{V} \\ 0, \text{ otherwise} \end{cases}$$

The n by ℓ by n tensor Δ is also established and D is mapped into Δ. This mapping process is shown in Figure 2-3.

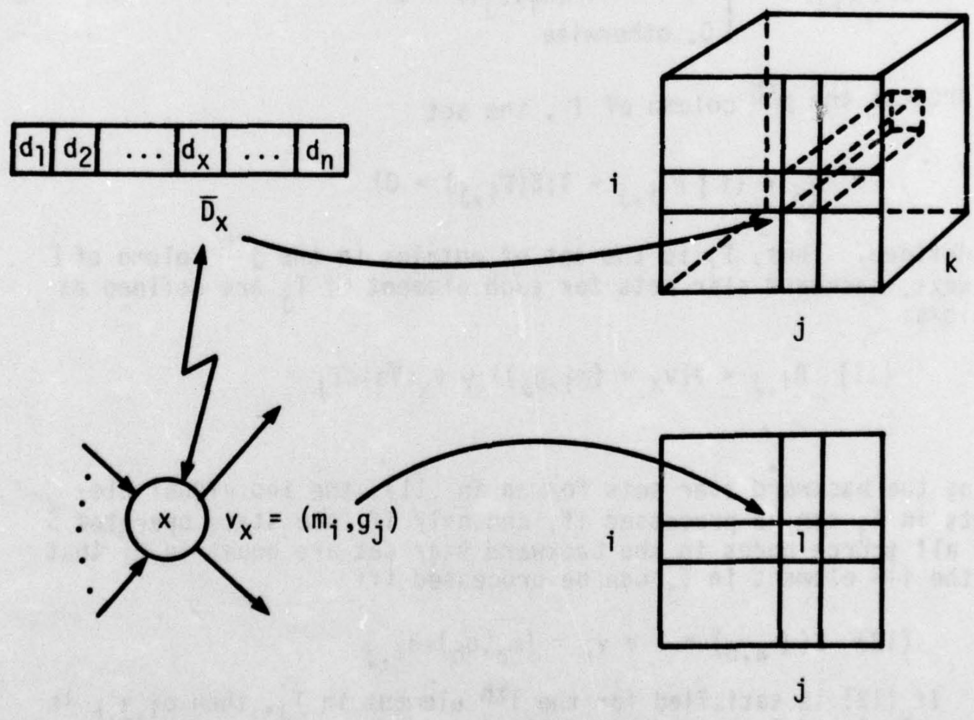

Figure 2-3. Mapping the Node Set to Γ and D to Δ

Since the elements in D are in one-to-one correspondence with the nodes in $\overline{V}$, it follows that an automorphism exists which maps Δ into Γ with respect to the first two indicies. The tensor Δ will be zero filled initially, and Γ will be used to determine the appropriate entries for Δ. Since the flows between nodes are left-to-right or up-or-down, Γ is processed by column from left-to-right, that is, column j of Γ is completely processed before processing column j+1. When processing is completed, $\Delta_{i,j,k}$ contains the percent of MOS i, grade j that originated in MOS k. Since the entries in Γ are surrogates for the node set, an entry $\Gamma_{i,j}$ cannot be processed until every node which sends flow to node $v = (m_i\ g_j)$ has been processed. To record such processing, the STATE OPERATOR S on Γ is defined such that:

$$S(\Gamma_{i,j}) = \begin{cases} 1, \text{ if } \Gamma_{i,j} \text{ has been completely processed} \\ 1, \text{ if } \partial\{v=(m_i,g_j)\} = \phi \\ 0, \text{ otherwise} \end{cases}$$

To process the $j^{\underline{th}}$ column of Γ, the set

$$T_j = \{i \mid \Gamma_{i,j} = 1; S(\Gamma_{i,j}) = 0\}$$

is defined. Thus, T_j is the set of entries in the $j^{\underline{th}}$ column of Γ. Next, backward star sets for each element of T_j are defined as follows:

$$(11) \quad B_{i,j} = \partial\{v_x = (m_i,g_j)\} \ \forall\ v_x \epsilon \overline{V} \ni i \epsilon T_j$$

Using the backward star sets formed in (11), the individual elements in T_j can be processed if, and only if, the state operator S for all source nodes in the backward star set are equal to 1; that is the $i^{\underline{th}}$ element in T_j can be processed iff

$$(12) \quad S(\Gamma_{a,b}) = 1 \ \forall\ v_x = (m_a,g_b) \epsilon B_{i,j}$$

If (12) is satisfied for the i^{th} element in T_j, then $\Delta_{i,j,k}$ is calculated for each k as follows:

$$(13) \quad \Delta_{i,j,k} = \Delta_{i,j,k} + p_{st}\, \Delta_{x,y,k}$$

$$\forall\ x,y \ni v_s = (m_x,g_y)\ \epsilon\ B_{i,j} \text{ for } k = 1,2,\ldots,n$$

When (13) has been completed for all source nodes and for all values of k, $\Gamma_{i,j}$ has been processed. The state operator $S(\Gamma_{i,j})$ is set to one and the set T_j is updated: $T_j = T_j - \{i\}$. If (12) is not satisfied, that element of T_j is bypassed and the next element of the T_j set is processed. If after sequentially examining and processing the elements in T_j, the set is null, then column j has been completely processed. If T_j is not null, the algorithm starts again with the first element in T_j and continues processing until T_j is null. Note that because of constraints (8) and (9), every pass through T_j reduces the cardinality of the set by at least 1. Since $|T_j|$ is finite, T_j must become null after a finite number of passes.

2-4. <u>Computing Interim MOS populations</u>. The Δ tensor has established an audit trail of movement of personnel between MOS and is now used to form interim MOS as if no change of MOS had occurred. For every MOS, define a tensor P such that $P^i_{y,g,q}$ is the strength of MOS i for year of service y, grade g and calendar quarter . A tensor I is also defined dimensioned by year of service, MOS and quarter such that $I_{y,m,q}$ is the strength of interim MOS m in year of service y in quarter q. The entries in I are computed as follows:

$$(14) \quad I_{y,m,q} = \sum_{i=1}^{n} \sum_{j=1}^{\ell} [(\Delta_{i,j,m})(P^i_{y,j,q})]$$

Note that (14) reallocates to interim MOS m everyone that originated in MOS m at the entry level. Having compensated for movement between MOS, interim MOS continuation rates are computed.

2-5. COMPUTING INTERIM MOS CONTINUATION RATES. Equations (15) and (16) illustrate two different methods for computing these rates. Let $\Psi_{m,y}$ be the continuation rate for interim MOS m in the yth year of service. One computational method is:

$$(15) \quad \psi_{m,y} = \frac{\sum_{q=5}^{8} I_{y+1,m,q}}{\sum_{q=1}^{4} I_{y,m,q}}$$

The other method is:

$$(16)\quad \psi_{m,y} = \frac{1}{4} \sum_{q=1}^{4} \frac{I_{y+1,m,q+4}}{I_{y,m,q}}$$

Both methods are heuristic; (15) is a weighted moving average and (16) is a simple moving average. Both use eight quarters of data and both are candidates for use; however, additional testing is required before a final selection is made.

2-6. MOS-UNIQUE CONTINUATION RATES. With the interim continuation rates, Δ can be used to form convex combinations of the interim MOS rates yielding continuation rates which are MOS-unique at the grade and year of service level of detail. The computational technique is given by (17) where $R_{i,j,y}$ is the rate for MOS i, grade j and year of service y.

$$(17)\quad R_{i,j,y} = \begin{cases} \sum_{k=1}^{n} (\Delta_{i,j,k})(\psi_{k,y}), & \text{iff } \Gamma_{i,j} = 1 \\ 0, & \text{otherwise} \end{cases}$$

CHAPTER 3
OBSERVATIONS

A measure of the validity of the methodology can be obtained by applying the rates to the data on which the rates are based. June 1975 and June 1976 data were used to compute Army wide continuation rates and MOS-unique continuation rates for the maneuver combat arms. Both rates were used to "predict" the 1976 first year of service population from the June 1975 data for MOS 11B, light weapons infantryman. The prediction obtained using MOS-unique rates was within 0.27% of actual experience while Army wide rates were in error by 69.5%. While this is a biased test, it appears that the use of MOS-unique rates offers a significant advantage over the current method of applying Army wide rates at the MOS level.

APPENDIX A

STUDY TEAM

1. STUDY TEAM

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   Ms Elizabeth A. Demshock

   Ms Joyce W. Garris

   Ms Nancy M. Lawrence

   Ms Linda L. Prieto

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APPENDIX B

1. Personnel Inventory Analysis/Years of Service/Objective Force Model, Technical Paper TP217, Systems Automation Corporation, Silver Spring, Maryland, 1974.

2. Rheinbolt, W., V. Basili, and C. Mesztenyi, On a Programming Language for Graph Algorithms, Technical Report TR 158, University of Maryland, Computer Science Center, College Park, Maryland, 1971.

3. AR 611-201, Enlisted Career Management Fields and Military Occupational Specialties.

APPENDIX C

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